The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2

Date: November 7, 2023 Course: EE 313 Evans

Name: ____
Last, First

- This in-person exam is scheduled to last 75 minutes.
- Open books, open notes, and open class materials, including homework assignments and solution sets and previous midterm exams and solutions.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network.
- ***Please disable all wireless connections on your calculator(s) and computer system(s).***
- Please mute all computer systems.
- Please turn off all phones.
- No headphones are allowed.
- All work should be performed on the midterm exam. If more space is needed, then use the backs of the pages.
- **Fully justify your answers.** If you decide to quote text from a source, please give the quote, page number and source citation**.**

Problem	*Point Value*	*Your score*	*Topic*
1	27		System Properties
2	24		Convolution
3	27		System Identification
4	22		Filter Design
Total	100		

Problem 2.1. *System Properties*. 27 points.

Each discrete-time system has input $x[n]$ and output $y[n]$, and $x[n]$ and $y[n]$ might be complex-valued.

Determine if each system is linear or nonlinear, time-invariant or time-varying, and bounded-input bounded-output (BIBO) stable or unstable.

You must either prove that the system property holds in the case of linearity, time-invariance, or stability, or provide a counter-example that the property does not hold. Providing an answer without any justification will earn 0 points.

Part	***System Name***	***System Formula***	***Linear?***	***Time-Invariant?***	***BIBO Stable?***
(a)	First-Order Difference Filter	$y[n] = x[n] - x[n-1]$ for $n \geq 0$ and $x[-1] = 0$			
(b)	Amplitude Modulation	$y[n] = x[n] \cos(\hat{\omega}_0 \, n)$ for $n \geq 0$ where $\hat{\omega}_0$ is a constant			
(c)	Exponentiation	$y[n] = e^{x[n]}$ for $-\infty < n < \infty$			

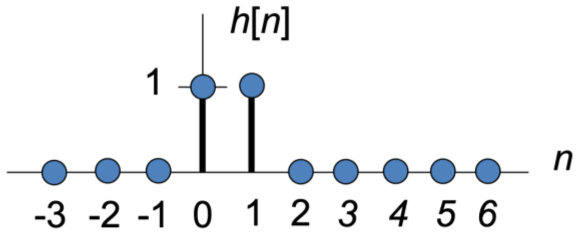
(a) First-Order Difference Filter: $y[n] = x[n] - x[n-1]$ for $n \geq 0$ and $x[-1] = 0$. *9 points*.

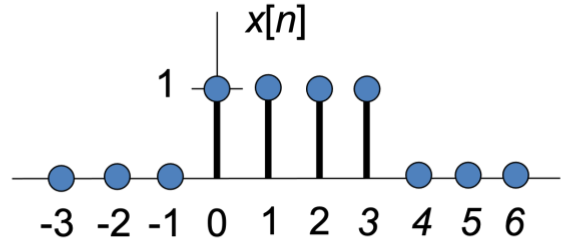
(b) Amplitude Modulation: $y[n] = x[n] \cos(\hat{\omega}_0 \, n)$ for $n \geq 0$ where $\hat{\omega}_0$ is a constant. *9 points*.

(c) Exponentiation: $y[n] = e^{x[n]}$ for $-\infty < n < \infty$. *9 points*.

Problem 2.2 *Convolution*. 24 points.

(a) Compute and plot $y[n] = h[n] * x[n]$ using the discrete-time rectangular pulses below. *12 points*.





(b) Compute and plot $y[n] = h[n] * x[n]$ using the discrete-time rectangular pulses below. *12 points*.

$$h[n] = \begin{cases} 1 & \text{for } 0 \le n \le L_h - 1 \\ 0 & \text{otherwise} \end{cases}$$

$$x[n] = \begin{cases} 1 & \text{for } 0 \le n \le L_x - 1 \\ 0 & \text{otherwise} \end{cases}$$

where $L_h < L_x$ and both L_h and L_x are positive integers. Give your answer in terms of L_h and L_x.

Problem 2.3 *System Identification.* 27 points.

You are given several causal discrete-time linear time-invariant (LTI) systems each with unknown impulse response but you are able to observe the input signal $x[n]$ and output signal $y[n]$ for $-\infty < n < \infty$.

For reference, the unit step function $u[n]$ is defined as

$$u[n] = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$y[n]$	$Y(z)$	Region of Convergence
$\delta[n]$	1	all z
$\delta[n-n_0]$	z^{-n_0}	$z \neq 0$
$u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
$a^n\, u[n]$	$\frac{1}{1-a\,z^{-1}}$	$\lvert z\rvert > \lvert a\rvert$

(a) When input is $x[n] = \delta[n] - \delta[n-1]$, output is $y[n] = \delta[n] - 2\,\delta[n-1] + \delta[n-2]$. Find the impulse response $h[n]$. *9 points*.

(b) When input is $x[n] = 0.9^n\, u[n]$, output $y[n] = \delta[n]$ where $\delta[n]$ is the discrete-time impulse:

$$\delta[n] = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{otherwise} \end{cases}$$

Find the impulse response $h[n]$. *9 points*.

(c) When the input is $x[n] = u[n]$, the output is $y[n]$ is a rectangular pulse of L samples in duration:

$$y[n] = \begin{cases} 1 & \text{for } 0 \leq n \leq L-1 \\ 0 & \text{otherwise} \end{cases}$$

Find the impulse response $h[n]$. *9 points*.

Problem 2.4. *Filter Design.* 22 points.

Consider designing discrete-time linear time-invariant (LTI) infinite impulse response (IIR) filters.

In this problem, **all the poles and zeros will be real-valued**.

In each part below, design a biquad by placing real-valued poles and zeros to achieve the indicated frequency selectivity (lowpass, highpass, bandpass, bandstop, allpass or notch) or indicate that no such biquad with real-valued poles and zeros could be designed.

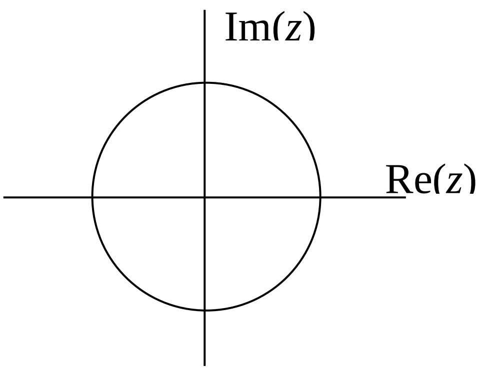
Please use O to indicate real-valued zero locations and X to indicate real-valued pole locations.

(a) ***A first-order LTI IIR filter*** has zero z_0 and pole p_0, and its transfer function in the z-domain of

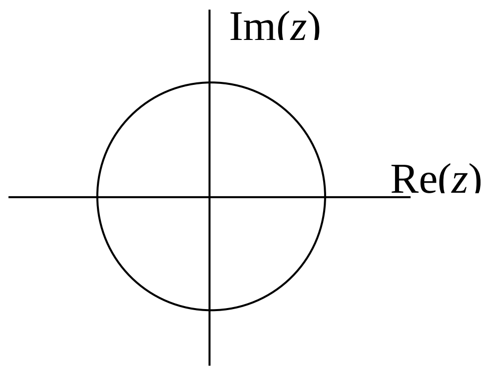
$$H(z) = C\,\frac{(z - z_0)}{(z - p_0)}$$

where C is a constant. Give numeric values for zero z_0 and pole p_0 to give each magnitude response below, place the zero and pole on the pole-zero diagram, and explain your reasoning. *10 points.*

(1) Lowpass filter



(2) Highpass filter

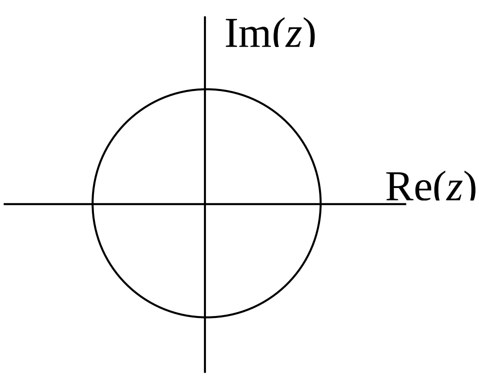


(b) ***A second-order LTI IIR filter*** has zeros z_0 and z_1 and poles p_0 and p_1, and its transfer function in the z-domain (where C is a constant) is

$$H(z) = C\,\frac{(z - z_0)(z - z_1)}{(z - p_0)(z - p_1)}$$

Give numeric values for zeros z_0 and z_1 and poles p_0 and p_1 to give each magnitude response below, place the zeros and poles on the pole-zero diagram, and explain your reasoning. *12 points.*

(3) Bandpass filter



(4) Bandstop filter